2206 Modeling Method of Stress Concentrated Members for First Order Analysis
- 3rd Report: Approximation of 3D-members -

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Abstract : This paper proposes a modeling method of stress concentrated members for the first order analysis(FOA). When a complicate solid structure is simplified as an assemblage of simple beam elements applied to element forces such as axial force, shear force, bending moment and twisting moment, the total stiffness equation for FOA of the solid structure is constructed and solved for the given boundary and loading conditions by the usual FEM manner. Then, the total deformation of simplified model is estimated and the element forces are calculated in the element level. The stress concentration of simplified each member with holes, notches and variable cross section is also predicted by the suggested approximate formulation. A U-notched 3D-model is picked up and how precisely the suggested formulas can predict the stress concentration will be shown. These formulas can be used to show the influence of the design parameter changes to the designer and how the load transmission path will change by the design changes.

Key Words: First Order Analysis, Stress Concentration Factor, Structural Optimization.

1. Introduction

Computer-aided engineering (CAE) has been widely accepted by industrial designers and simulation engineers, and the structural optimization software such as GENESIS is frequently used for the design optimization that satisfies the specification by modeling the detailed structural elements. However it is difficult for the designers to interpret why the final design obtained by the software is optimum, and no design knowledge is accumulated in the designer as skillful engineering know-how. To overcome this problem, a new concept of design tool named First Order Analysis (FOA) has been invented to treat the skeletal structures which consist of beams and panels such as car body[1)]. The developed FOA tool provides a useful aid to design of the load transmission path and topology optimization of skeletal structures at upstream stage of design processes, because the tool includes a variety of graphic interfaces using MS Excel for the design engineers and powerful topology optimization module.

However the FOA proposed by Toyota Central R&D lab[1] includes only beam and panel elements as design element, it is not enough to simplify the design model of complicate solid structures such as machine tools in all kinds of structural design optimization. Moreover, the suggested FOA can treat only the rigidity of the body structure. When we consider the optimum design of continuum structures in the upstream design process, the stress concentration as well as the structural rigidity should be taken in to account in the design process and the FOA should be extended to provide such ability for the designers. The objective of this research is to develop a new FOA concept, which can be applied to the structural design analysis and optimization of general solid structures considering the deformation/rigidity of the structure as well as stress concentration. The main objectives of FOA for continuum structure are: 1) to be able to predict total deformation and load transmission as well as stress concentration of complicate structures by the simplified model, and 2) to be able to assist the design engineers to optimize the structure, to make them understandable how the load is transmitted rationally to the support, and which parameter is important for optimization of structures.

In the practical cases, the design engineers are expected creative design products, and they have to check the possibility of many kinds of structural candidates of very complicate machine elements. To realize this design process and to assist them for creative and interactive thinking on the computer, software embedded the following mentioned steps with graphical interface is expected to develop. To achieve such kind of design environment for the engineer, it is required to prepare the element stiffness library and stress concentration factor library as functions of the design parameters and the element forces for the many kinds of elements with many kinds of notches, defects, holes and variable cross sections.

To develop formulas of stress concentration factor prediction, we have suggested the estimation flow of new FOA and have shown how to predict the stress concentration of complicate members in the first report[2)]. In this report, we select a U-notch in a cylinder for 3-dimensional cases, which are taken as examples among the different many kinds of element shapes, applied tensile, bending or shear loading at the ends. The formulation will be shown briefly in the next section. By using the same way we can prepare formulas for other shape of the model like solids with other kind of notch, holes or any defects.

2. Construction of FOA for Continuum Structures

For a complicated continuum structure it is difficult to understand how the load is transmitted to the support, and the influence of change of design variables to the total deformation or stress concentration factor. So to satisfy these requirements, the FOA for the continuum structures is constructed by the following steps:

(1) Simplify the whole complex model as an assemblage of a various kinds of simple elements. Figs.1 (a) and (b) show the total model and its simplified model as the assemblage of simple elements with wholes, notches and step changes of cross sections.

(2) Decompose each element to a simple element applied forces such as axial and shear forces, bending moments and twisting. If the relationship between element stiffness and the design parameters is prepared for a various kinds of simplified elements in the computer library, we may invoke the stiffness matrix for the specified design parameters.

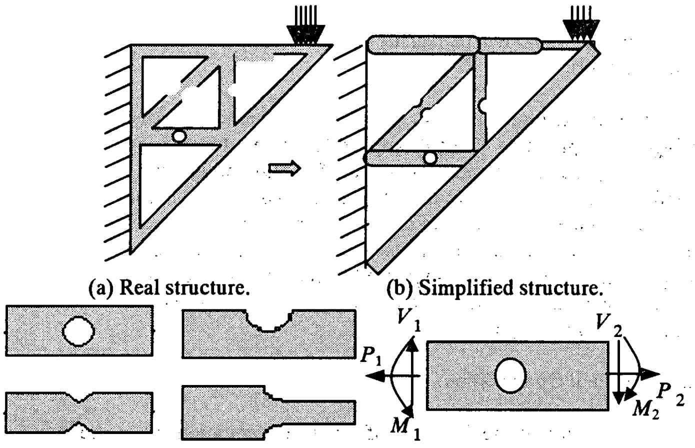
(3) Construct the total stiffness equation as an assemblage of the simple elements. For example, if a beam element with a V-notch with the design parameters such as width W, a half of the distance between the two notch root a, root radius of notch ρ, and notch angle α as shown in Fig.1(c), the element stiffness equation is given as

$$\mathbf{F}_e = \mathbf{K}_e \, \mathbf{U}_e , \tag{1}$$

where $\mathbf{F}_e = (P_1, V_1, M_1, P_2, V_2, M_2)^T$, $\mathbf{U}_e = (u_1, v_1, \theta_1, u_2, v_2, \theta_2)^T$ when P_i, V_i, M_i (i = 1, 2) denote axial force, shear force and bending moment, and u_i, v_i, θ_i (i = 1, 2) denote the axial and vertical displacement, and rotation, respectively. The stiffness matrix of simple element should be prepared as a function of the design parameters as $\mathbf{K}_e = \mathbf{K}_e(\rho, a, W, \alpha)$ in advance. Then, the element stiffness matrix is assembled (summed) to obtain the total stiffness matrix and thereby solved for the total displacement of the structure under the given loading and boundary conditions.

(4) Calculate the element forces to predict the stress concentration for the given design parameters. To do so, some sophisticated formulas are prepared for the maximum stress $\sigma_{max} = \sigma_{max}(\rho, a, W, \alpha)$ in advance, which are described later in detail.

(5) Display the deformation and element forces. Then, the designer can observe how the load is transmitted, and also the stress concentration factor at each element.
(6) Change the structural topology if needed and optimize the design parameters.



(a) Real structure. (b) Simplified structure.

(c) Element library. (d) Forces of element with a hole

Fig.1 Decomposition of 2-dimensional continuum structure.

If we have to change any dimension or shape like as preparing the element with both side notch, one side notch, or holes, then we have to go to step 2, or to step 1 if required the structural model change.

3. Polynomial Regression

Approximating model is based on observed data and normally illustrated in a polynomial function. The polynomial function of second-order response surface model with two variables are

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 + \varepsilon \quad (2)$$

where β_i (i = 1,..., 5) are regression coefficients and ε is a random error. Let $x_1^2 = x_3, x_2^2 = x_4, x_1 x_2 = x_5$, then Eq.(2) becomes

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \varepsilon . \quad (3)$$

Suppose there are k regression variables, then

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \text{-------------} + \beta_k x_k + \varepsilon \quad (4)$$

This linear model may be written in matrix notation as

$$\mathbf{Y} = \mathbf{X}\mathbf{b} + \mathbf{e}. \quad (5)$$

In general, **Y** is a (n*1) vector of the observations, **X** is a (n*k) matrix of the levels of the independent variables, **b** is a (k*1) vector of the regression coefficients, and **e** is an (n*1) vector of random errors, where n is the number of observations.

The method of least squares requires minimizing the scalar sum of squares

$$\mathbf{e}^T \mathbf{e} = (\mathbf{Y}-\mathbf{X}\mathbf{b})^T (\mathbf{Y}-\mathbf{X}\mathbf{b}) = \mathbf{Y}^T\mathbf{Y} - \mathbf{b}^T\mathbf{X}^T\mathbf{Y} - \mathbf{Y}^T\mathbf{X}\mathbf{b} + \mathbf{b}^T\mathbf{X}^T\mathbf{X}\mathbf{b} \quad (6)$$

Minimizing the Eq.(6), we get the square estimator of the vector

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \quad (7)$$

then, fitted regression model is

$$\hat{\mathbf{Y}} = \mathbf{X}\,\mathbf{b} \quad (8)$$

4. Formulation of Stress Concentration Prediction

Now a day it is widely recognized that the stress concentration affects the lifetime of machine products under tension, bending, and/or shear, and that the stress concentration factor is of paramount importance for assessing fatigue strength in notched machine parts.

Here a cylinder with deep U-notch shown in Fig.2 is considered. An approximate formula is derived as a function of design variables. The diameter of the cylinder D is kept constant, and the ratios of U-notch depth to the diameter $x_1 = 2d/D$ and root radius to the diameter $x_2 = 2\rho/D$ are considered as the parameters (design variables). The tensile and shear forces, and bending moment as well as twisting moment are applied at the ends, and the upper and lower bounds (changeable ranges) are assigned for each design parameters as $0.200 \leq x_1 \leq 0.400$, $0.030 \leq x_2 \leq 0.150$.

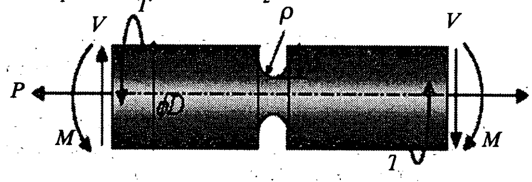


Fig. 3 A solid bar with U-notch.

To get observation matrix **X**, 9 sampling points of design parameter combination for $2d/D$ and $2\rho/D$ are analyzed, and the stress concentration factor for tensile, bending, twisting and shear forces are calculated by keeping all other parameters fixed. Then, we get maximum stress occurs at the notch portion, and the following approximated formulas are obtained.

Tensile force:

$$\sigma_{max}/\sigma_a = 3.4819 + 12.4958x_1 - 11.8754x_2 - 16.6550x_1^2 + 10.8181x_2^2 - 39.5939x_1x_2 + \varepsilon \quad (9)$$

Bending force:

$$\sigma_{max}/\sigma_b = 2.9006 + 6.9710x_1 - 8.4533x_2 - 12.0235x_1^2 - 16.9534x_2^2 - 9.8178x_1x_2 + \varepsilon \quad (10)$$

Shearing force:

$$\tau_{max}/\tau_a = 3.4689 + 2.1287x_1 - 11.3619x_2 - 8.1378x_1^2 - 5.7140x_2^2 + 9.2948x_1x_2 + \varepsilon \quad (11)$$

Twisting force:

$$\tau_{max}/\tau_b = 1.9389 + 3.4569x_1 - 1.1190x_2 - 4.8962x_1^2 - 3.9723x_2^2 - 17.8172x_1x_2 + \varepsilon \quad (12)$$

where nominal stresses are $\sigma_a = P/\pi r^2$ for tensile force P, $\sigma_b = 4M/\pi r^3$ for bending moment M, $\tau_a = V/\pi r^2$ for shear force V, and $\tau_b = 2T/\pi b^3$ for twisting moment T, in which r denotes radius of the cylinder at notch portion of minimal cross section.

To prove the reliability and effectiveness, the derived formulas have been applied for the case $2d/D$ = 0.225 and $2\rho/D$ = 0.1 when the nominal stress ratio of tension to bending moment, σ_a/σ_b = 1.937, are applied, and the nominal stress ratio of tension to twisting moment, σ_a/τ_b = 3.875, are applied. Then, the maximum stress at the notch portion is predicted from our formula as σ_{max}/σ_a = 8.347, whereas the direct CAE analysis gives σ_{max}/σ_a= 7.80, and the error is 7.01%. It is found that prediction by suggested formulas is reliable and useful for the FOA of complicate continuum structures.

5. Conclusions

In this report, a new FOA process for the complicate solid structures is proposed, and the approximate formulas of stress concentration factor of element with notch have been described, which will be helpful for design purposes. It has been confirmed the validity of suggested method numerically

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